5.3 Inverse Functions & Their Derivatvies
5.4 Derivatives & Integrals of e^x

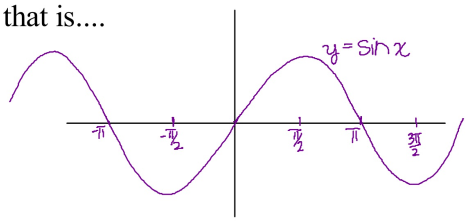
ESSENTIAL QUESTIONS:

1. How can we find the derivative of $f^{-1}(x)$ at $x = a$, without actually finding $f^{-1}(x)$?

2. What is unique about the derivative and the integral of $y = e^x$?

Let's review what we know about the inverse of a function f.

1. f(x) must be one-to-one to have an inverse. If f(x) is not one-to-one, define an interval that is....



A function is one-to-one if it is strictly monotonic. This means that the function is either increasing everywhere on its domain or decreasing everywhere on its domain.

How can we show this for a given function?
Look at the sign of the derivative:

$f'(x) > 0 \Rightarrow f$ is increasing
$f'(x) < 0 \Rightarrow f$ is decreasing

Show that $f(x) = 2 - x - x^3$ is strictly monotonic.

To find the derivative of f^{-1} (without finding f^{-1}):

$$\left(f^{-1}\right)'(x) = \frac{1}{f'[f^{-1}(x)]}$$

for some $x = a$

1. Find $f^{-1}(a)$.
2. Find $f'(x)$.
3. Substitute into $\left(f^{-1}\right)'(x) = \frac{1}{f'[f^{-1}(x)]}$

Example:
Find $(f^{-1})'(-2)$ if $f(x) = 2x^5 + x^3 + 1$.

A function and its derivative take on the values shown in the table. Let $g(x) = f^{-1}(x)$. Find $g'(6)$.

x	f(x)	f'(x)
2	6	$\frac{1}{2}$
6	8	$\frac{1}{3}$

DERIVATIVES of e^x

If $y = e^x$, then $\frac{dy}{dx} = e^x$.

If u is a function of x

and $y = e^u$, then $\frac{dy}{dx} = u'e^u$.

Find the derivatives:

1. $y = e^{x^2}$

2. $y = 2xe^{-x}$

3. $y = e^{2\ln(4x)}$

4. Find the equation of the line tangent to $y = e^{3x}$ at $x = 0$.

INTEGRALS of e^x

$$\int e^x \, dx = e^x + C$$

If u is a function of x, then

$$\int e^u \, du = e^u + C$$

Find each integral.

1. $\int e^{1-x} \, dx$

2. $\int \frac{e^{2x}}{1+e^{2x}} \, dx$

3. $\int e^{\tan 2x} \sec^2 2x \, dx$